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Power Counting and Effective Field Theory for Charmonium

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Abstract

We hypothesize that the correct power counting for charmonia is in the parameter $\Lambda_{\rm QCD}/m_c$, but is not based purely on dimensional analysis (as is HQET). This power counting leads to predictions which differ from those resulting from the usual velocity power counting rules of NRQCD. In particular, we show that while $\Lambda_{\rm QCD}/m_c$ power counting preserves the empirically verified predictions of spin symmetry in decays, it also leads to new predictions which include: A hierarchy between spin singlet and triplet octet matrix elements in the J/ψ system. A quenching of the net polarization in production at large transverse momentum. No end point enhancement in radiative decays. We discuss explicit tests which can differentiate between the traditional and new theories of NRQCD.

I. INTRODUCTION

Quarkonia have proven to be fruitful in helping us gain a better understanding of QCD. For large enough valence quark masses the system should be dominated by Coulomb exchange in the perturbative regime. Fortunately, the physical valence quark masses seem to be too small for the states to be truly insensitive to non-perturbative effects, and thus give a window on the more interesting aspects of QCD. In order to systematically study these effects we need to separate the long distance from the short distance physics. This can be accomplished by writing down a proper effective field theory to describe the infra-red. The theory should provide a power counting which determines which operators are relevant. In most effective theories this power counting is based upon dimensional analysis. However, for non-relativistic QCD (NRQCD) [1] this is not the case. Instead, it is an expansion in the parameter v, the relative velocity of the valence quarks. This power counting presupposes that the states are Coulombic, at least to the extent that $\alpha_s(mv) \simeq v$, and leads to the result that operators of the same dimension may be of different orders in the power counting. This methodology has been applied to the J/ψ as well as the Υ systems. While it seems quite reasonable to apply this power counting to the Υ system, it is not clear, as we will discuss in more detail below, that it should apply for the J/ψ system. Indeed, we believe that the data is hinting towards the possibility that a new power counting is called for in the charmed system.

In Ref. [1], the authors showed how to utilize NRQCD to predict decay rates as well as production rates in a systematic double expansion in α_s and v. These predictions have met with varying degrees of success. For instance, it is possible to explain J/ψ and ψ' production at the Tevatron, though the initial data on the polarization of these states at large transverse momentum [2] seems to be at odds with the NRQCD prediction. In addition, there is an unexpected hierarchy of matrix elements in the charmed system which does not seem to be there in the bottom system. Furthermore, there is a discrepancy between theoretical expectations and data for the end point spectrum of inclusive radiative decays. While we do not believe that any one of these pieces of evidence, on their own, is strong enough to warrant introduction of a new theory, it seems to us that the evidence, taken as a whole, seems to be telling us that the effective field theory which best describes the J/ψ system may not be the same theory which best describes the Υ. The purpose of this paper is to present an alternative charmonium power counting, first discussed in [3] and later utilized to study the quark-antiquark potential [4], which leads to predictions which seem to have better agreement with the data.

II. BACKGROUND

A general decay process may be written in factorized form [1]

$$\Gamma_{J/\psi} = \sum C_{^{2S+1}L_J}(m, \alpha_s) \langle \psi \mid O^{(1,8)}(^{2S+1}L_J) \mid \psi \rangle. \tag{2.1}$$

The matrix element represents the long distance part of the rate and may be thought of as the probability of finding the heavy quarks in the relative state n, while the coefficient $C_{^{2S+1}L_J}(m, \alpha_s)$ is a short distance quantity calculable in perturbation theory. The sum over operators may be truncated as an expansion in the relative velocity v.

Similarly, production cross sections may be written as

$$d\sigma = \sum_n d\sigma_{i+j\to Q\bar{Q}[n]+X} \langle 0 \mid \mathrm{O}_n^H \mid 0 \rangle. \tag{2.2}$$

Here $d\sigma_{i+j\to Q\bar{Q}[n]+X}$ is the short distance cross section for a reaction involving two partons, i and j, in the initial state, and two heavy quarks in a final state, labeled by n, plus X. This part of the process is calculable in perturbation theory, modulo the possible structure functions in the initial state. The production matrix elements, which differ from those used in the decay processes, describe the probability of the short distance pair in the state n to hadronize, inclusively, into the state of interest. The relative size of the matrix elements in the sum are again fixed by the power counting which we will discuss in more detail below.

The formalism for decays is on the same footing as the operator product expansion (OPE) for non-leptonic decays of heavy quarks, while the production formalism assumes factorization, which is only proven, and in some applications of production this is not even the case, in perturbation theory [5]. The trustworthiness of factorization depends upon the particular application, as we will discuss further in the body of the paper. We have reviewed these results here to emphasize the point that when we test this theory we are really testing both the factorization hypothesis as well the validity of the effective theory as applied to the J/ψ system. Thus, we must be careful in assigning blame when we find that our theory is not agreeing with the data.

While NRQCD has allowed for successful fits of the data (in particular we have J/ψ and ψ' production at the Tevatron in mind), its predictive power has yet to stand any stringent test.[1] Indeed, one robust prediction of the theory, namely that production at large transverse momentum is almost purely polarized [7–9], seems to be at odds with the initial data.[2] Other predictions such as the ratio of χ_1/χ_2 in fixed target experiments and the photon spectrum in inclusive radiative decays also seem to disagree with the data, as we shall discuss in more detail below. We are left with two obvious possibilities: 1) The power counting of NRQCD does not apply to the J/ψ system. 2) Factorization is violated "badly", meaning that there are large power corrections. The purpose of this paper is to explore the first possibility.

If we assume that NRQCD does not apply to the J/ψ system, then we must ask: is there another effective theory which does correctly describe the J/ψ? One good reason to believe that such a theory does exist is that NRQCD, as formulated, does correctly predict the ratios of decay amplitudes for exclusive radiative decays. Using spin symmetry the authors of [7] made the following predictions:

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[1]One simple test, which has yet to be performed, is to compare the values of the decay and production singlet matrix elements, which are predicted to be equal at leading order in v [1]. To date the production singlet matrix elements have yet to be extracted and compared to the decay singlet matrix elements. These extractions can easily done using the direct J/ψ production data at CLEO [6].

[2]The data still has rather large error bars, so we should withhold judgment until the statistics improves.

$$\Gamma(\chi_{c0} \to J/\psi + \gamma) : \Gamma(\chi_{c1} \to J/\psi + \gamma) : \Gamma(\chi_{c2} \to J/\psi + \gamma) : \Gamma(h_c \to \eta_c + \gamma)$$
$$= 0.095 : 0.20 : 0.27 : 0.44 \quad (theory)$$
$$= 0.092 \pm 0.041 : 0.24 \pm 0.04 : 0.27 \pm 0.03 : unmeasured \quad (experiment). \qquad (2.3)$$

Thus, we would like to find an alternative formulation (power counting) of NRQCD which preserves these predictions yet yields different predictions in other relevant processes. Before discussing this alternative power counting, we must briefly review the standard formulation.

The rest of this paper is structured as follows. We will first review the standard power counting used for predictions to date. Then we will offer a new power counting and discuss how the two theories differ in their treatment of several relevant observables, as well as how the theories fair against the data. We close with some remarks regarding the validity of factorization in various observables.

A. NRQCD power counting

The power counting depends upon the relative size of the four scales ($m, mv, mv^2, \Lambda_{\rm QCD}$). If we take $m > mv > mv^2 \simeq \Lambda_{\rm QCD}$, then the bound state dynamics will be dominated by exchange of Coulombic gluons with ($E \simeq mv^2, \vec{p} = m\vec{v}$). This hierarchy has been assumed in the NRQCD calculation of production and decay rates and is most probably the reasonable choice for the Υ system, where $mv \sim 1.5$ GeV. However, whether or not it is correct for the J/ψ, where $mv \sim 700$ MeV remains to be seen.

The power counting can be established in a myriad of different ways. Here we will follow the construction of [10], which we now briefly review. There are three relevant gluonic modes [11]: the Coulombic (mv^2, mv), soft (mv, mv) and ultrasoft (mv^2, mv^2). The soft and Coulombic modes can be integrated out leaving only ultrasoft propagating gluons. In the process of integrating out these modes we must remove those large modes from the quark field. This is accomplished by rescaling the heavy quark fields by a factor of $\exp(i\vec{p}\cdot\vec{x})$ and labeling them by their three momentum $\vec{p}$. The ultrasoft gluon can only change residual momenta and not labels on fields. This is analogous to HQET, where the four-velocity labels the fields and the non-perturbative gluons only change the residual momenta [12]. This rescaling must also be done for soft gluon fields [13] which, while they cannot show up in external states, do show up in the Lagrangian.[3] After this rescaling a matching calculation leads to the following tree level Lagrangian [10]

$$\begin{aligned}\mathcal{L} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left\{ iD^0 - \frac{(\mathbf{p})^2}{2m} \right\} \psi_{\mathbf{p}} - 4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'} \Bigg\{ \frac{1}{q^0} \psi^\dagger_{\mathbf{p}'} \left[A^0_{q'}, A^0_q \right] \psi_{\mathbf{p}} \\ + \frac{g^{\nu 0}\left(q'-p+p'\right)^\mu - g^{\mu 0}\left(q-p+p'\right)^\nu + g^{\mu\nu}\left(q-q'\right)^0}{(\mathbf{p'}-\mathbf{p})^{\mathbf{2}}} \psi^\dagger_{\mathbf{p}'} \left[A^\nu_{q'}, A^\mu_q \right] \psi_{\mathbf{p}} \Bigg\} \\ + \psi \leftrightarrow \chi,\ T \leftrightarrow \bar{T} + \sum_{\mathbf{p},\mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^{\mathbf{2}}} \psi^\dagger_{\mathbf{q}} T^A \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{q}} \bar{T}^A \chi_{-\mathbf{p}} + \ldots\end{aligned} \qquad (2.4)$$

[3] Thus the nomenclature is slightly misleading since we have not removed these fields from the Lagrangian.

where we have retained the lowest order terms in each sector of the theory. The matrices T^A and $\bar{T}^A$ are the color matrices for the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations, respectively. Notice that the kinetic piece of the quark Lagrangian is just described by a label. This is a result of the dipole expansion [14] which is used to get a homogeneous power counting. The last term is the Coulomb potential, which is leading order and must be resummed in the four-quark sector, while the other non-local interactions arise from soft gluon scattering.

Now all the operators in the Lagrangian have a definite scaling in v. The spin symmetry, which will play such a crucial role in the polarization predictions, is manifest. The two subleading interactions which will dominate our discussion are the "electric dipole" ($E1$)

$$\mathcal{L}_{E1} = \psi_{\mathbf{p}}^\dagger \frac{\vec{p}}{m} \cdot \vec{A} \psi_{\mathbf{p}}, \tag{2.5}$$

and "magnetic dipole" ($M1$)

$$\mathcal{L}_{M1} = c_F \, g \psi_{\mathbf{p}}^\dagger \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m} \psi_{\mathbf{p}}. \tag{2.6}$$

The $E1$ interaction is down by a factor of v while the $M1$ is down by a factor of v^2. The extra factor of v stems from the fact that the magnetic gluons are ultrasoft,[4] and the derivative operator therefore picks up a factor of v^2. These operators play a crucial role in the so-called octet mechanism.

B. New Power Counting

Let us now consider the alternate hierarchy $m > mv \sim \Lambda_{\rm QCD}$. One might be tempted to believe that in this case the power counting should be along the lines of HQET, where the typical energy and momentum exchanged between the heavy quarks is of order $\Lambda_{\rm QCD}$. However, this leads to an effective theory which does not correctly reproduce the infra-red physics. With this power counting, the leading order Lagrangian would simply be

$$\mathcal{L}_{\rm HQET} = \psi_v^\dagger D_0 \psi_v, \tag{2.7}$$

where the fields are now labeled by their four velocity. This is a just a theory of time-like Wilson lines (static quarks) which does not produce any bound state dynamics. Thus we are forced to the conclusion that the typical momentum is of order $\Lambda_{\rm QCD}$, whereas the typical energy is $\Lambda_{\rm QCD}^2/m$. The dynamical gluons are now all of the type $(\Lambda_{\rm QCD}, \Lambda_{\rm QCD})$, as the on-shell ultrasoft modes get cut-off by the confinement scale. Therefore, one no longer labels fields by their three velocities. The only label is the four velocity of the heavy quark. However, the $D^2/(2m)$ is still relevant and their is no dipole expansion. We can not resist

[4]One may wonder why the emission of a soft gluon cannot lead to the enhancement of the magnetic transition operator. However, the emission of such a gluon leaves the quark off-shell and contributes a pure counter-term to the matching [3].

the temptation of introducing yet another acronym,[5] and call this theory NRQCD$_c$, while we will refer to the traditional power counting as NRQCD$_b$ as we assume that it does describe the bottom system.[6]

The power counting of this theory is now along the lines of HQET where the expansion parameter is $\Lambda_{\rm QCD}/m_Q$. However the residual energy of the quarks is order $\Lambda^2_{\rm QCD}/m_Q$, while the residual three momentum is $\Lambda_{\rm QCD}$. Thus one must be careful in the power counting to differentiate between time and spatial derivatives acting on the quark fields. As far as the phenomenology is concerned, perhaps the most important distinction between the power counting in NRQCD$_c$ and NRQCD$_b$ is that the magnetic and electric gluon transitions are now of the same order in NRQCD$_c$. This difference in scaling does not disturb the successes of the standard NRQCD$_b$ formulation but does seem help in some of its shortcomings.

III. LIFETIMES

In the case of inclusive decays the use of effective field theory put theoretical calculations on surer footing. Previous to the advent of NRQCD, inclusive decays were written as a product of a short distance decay amplitude and a long distance wave function which was usually taken from potential models [17]

$$\Gamma_{J/\psi} = \mid \psi(0) \mid^2 C(m, \alpha_s). \tag{3.1}$$

Most of the time this formalism is adequate, however there is the question of the scheme dependence of the potential wave function beyond leading order. Beyond this drawback is the question of how to factor infra-red divergences in P wave decays. Within the effective field theory approach, however, these issues are clarified. The rate is now written as

$$\Gamma_{J/\psi} = \sum C_{^{2S+1}L_J}(m, \alpha_s) \langle \psi \mid O_{(1,8)}(^{2S+1}L_J) \mid \psi \rangle. \tag{3.2}$$

The operator matrix element gives the probability to find the quarks within the hadron in the state $^{2S+1}L_J$. The quarks can be either in a relative singlet or octet state, hence the subscript $(1,8)$. The matrix elements are well defined scheme dependent quantities, which can be measured on the lattice, or extracted from the data [18], and have definite scalings in v. For instance, consider the operator $\langle \chi_J \mid O_1(^3P_J) \mid \chi_J \rangle$. We would expect this operator to dominate the decay of χ states, given that the quantum number of the short distance quark pair match the quantum number of the final state. However, this is not the case [19]. The operator $\langle \chi_J \mid O_8(^3S_1) \mid \chi_J \rangle$ is of the same order. This can be seen from the fact that

[5] We stole this bit of prose from [15].

[6] In the language of [16], NRQCD$_b$ would correspond to pNRQCD and NRQCD$_c$ would correspond to NRQCD. We chose to introduce these new acronyms because calling NRQCD$_c$ NRQCD would be misleading, since the original NRQCD, as defined in [1], is indeed distinct from NRQCD$_c$. We thus believe that our labeling will be the simplest for our purposes and hope the community will indulge us in our, what may be perceived as gratuitous, acronymization.

the P wave operator comes with two spatial derivatives. The octet S operator vanishes at leading order, since there is no S wave component in the leading order hadronic state in the effective theory. The first non-vanishing contribution comes from two insertion of $E1$ operators into time ordered products with $O_8(^3S_1)$. Thus both the singlet P wave operator and the octet S wave operator scale as v^2. Furthermore, the inclusion of this operator into the rate allows for the proper absorption of infrared divergences in the P wave decays into octet S wave matrix elements. This should be considered a formal success of the effective field theory. Any change in the power counting will not change this success, as the scaling of an operator is independent of its renormalization group properties. Such a change could only effect the relevance of the infra-red divergence, in a technical sense.

The advent of NRQCD had little impact on the phenomenology of inclusive decays because it simply justified previous calculations of the total width. However, one novel prediction of NRQCD was found in the end point spectrum of inclusive radiative decays [20,21]. Radiative decays, as opposed to hadronic decays,[7] have the advantage that they are subject to an operator product expansion, thus rely less upon local-duality assumptions. The integration over the photon energy smears through resonances and thus one may expect the prediction to be more trustworthy. We may reliably calculate the photon spectrum itself if we smear over regions of phase space which are larger than $\Lambda_{\rm QCD}$ [20].

In NRQCD, the decay of the J/ψ is dominated by the $^3S_1$ singlet operator with the octet operators being suppressed by $v^4\pi/\alpha_s$. However, due to the singular nature of the octet Wilson coefficients (it is a delta function at leading order in α_s) they can become leading order near the end point of the spectrum. Of course we do not expect a delta function spike at the end point since the spike should be smeared out due to bound state dynamics, among other effects [22], which can be taken into account by the introduction of structure functions.

In [20] it was shown that if one smeared the photon spectrum over a range of order mv^2, then the spectrum would receive a leading order corrections from the octet matrix elements which are peaked at the end point. In the standard hierarchy such a smearing is satisfactory since it corresponds to smearing over mv in hadronic mass which is larger than $\Lambda_{\rm QCD}$. However, in the new power counting this is no longer true and, given that the OPE breaks down in the region where the octet was suppose to dominate, it is no longer true that we can predict any peak with reliability. If we now consider the data, we see that for the J/ψ the data is monotonically decreasing [23]. On the other hand the Υ data does show a bump out at larger values of the photon energy. We wish to take this as support for the new power counting in the J/ψ system. But we must be careful since there are other effects which become important at the end point which we have not taken into account. For instance, near the end point there are large radiative corrections which are known to resum into a Sudakov suppression. However, we would not expect this effect to completely eliminate the bump, just cut it off at larger energies. Nonetheless a complete calculation of the resummed Sudakov effects in the end point spectrum of Υ decays is needed.

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[7] We are ignoring photon fragmentation for the moment.

IV. HADRO-PRODUCTION

As discussed in section II a general production process may be written in the factorized form (2.2). The long distance part of the process involves the hadronization of the heavy quarks in the state n into the hadron of choice H. The matrix element in Eq. (2.2) is written as

$$\begin{aligned}\langle 0 \mid O_n^H \mid 0\rangle &= \langle 0 \mid \psi^\dagger \Gamma^{n'} \chi \mid \sum_X H + X\rangle\langle H + X \mid \chi^\dagger \Gamma^n \psi \mid 0\rangle \\ &\equiv \langle O_n^H\rangle.\end{aligned} \tag{4.1}$$

The tensor Γ^n operates in color as well as spin space and also contains possible derivatives. This tensor determines the order of the matrix element. If the quantum numbers n do not match the quantum numbers of the hadron, then the matrix element vanishes, as the hadronic states are those of the effective field theory, and are pure in the sense of a Fock space expansion. To get a non-vanishing result one must insert subleading operators into a time ordered product with the operator O_n^H. The number and order of the inserted operators determine the scaling of the matrix element, as we detail below.

A. Collider experiments

The leading order contribution to ψ production in the original v power counting scheme is through the color-singlet matrix element $\langle O_1^\psi(^3S_1)\rangle$, since the quantum numbers of the short distance quark pair matches those of the final state. All other matrix elements need the insertion of operators into time ordered products to give a non-zero result. Unlike the case of the χ discussed earlier, all other matrix elements are suppressed compared to the color-singlet matrix element above. For instance, the matrix element $\langle O_8^\psi(^1S_0)\rangle$ vanishes at leading order. The first non-vanishing contribution comes from the insertion of two $M1$ operators into time ordered products, thus giving a v^4 suppression. The scalings of the relevant matrix elements for ψ production in NRQCD$_b$ are shown in Table I. It appears from just the v counting that only the color-singlet contribution is important. But the other contributions can be enhanced for kinematic reasons. At large transverse momentum, fragmentation type production dominates [24], and only the $\langle O_8^\psi(^3S_1)\rangle$ contribution is important. Without the color-octet contributions (*i.e.*, the Color-Singlet Model), the theory is below experiment by about a factor of 30. By adding the color-octet contribution the fit to the data is very good [25]. The new power counting must also reproduce this success.

The relative size of the different matrix elements change in NRQCD$_c$. In particular, the $M1$ transition is now the same order as the $E1$ transition. The new scalings are shown in Table I [26]. Due to the dominance of fragmentation at large transverse momentum, we need to include effects up to order $(\Lambda_{\rm QCD}/m_c)^4$, since the $\langle O_8^\psi(^3S_1)\rangle$ matrix element will still dominate at large p_T.

Is this consistent? The size of the matrix elements is a clue. Extraction of the matrix elements uses power counting to limit the number of channels to include in the fits. Calculating J/ψ and ψ' production up to order $(\Lambda_{\rm QCD}/m_c)^4$ in NRQCD$_c$ requires keeping the same matrix elements as in NRQCD$_b$. Previous extractions of the matrix elements only involve the linear combination

	$\langle O_1^\psi(^3S_1)\rangle$	$\langle O_8^\psi(^3S_1)\rangle$	$\langle O_8^\psi(^1S_0)\rangle$	$\langle O_8^\psi(^3P_0)\rangle$
NRQCD$_b$	v^0	v^4	v^4	v^4
NRQCD$_c$	$(\Lambda_{\rm QCD}/m_c)^0$	$(\Lambda_{\rm QCD}/m_c)^4$	$(\Lambda_{\rm QCD}/m_c)^2$	$(\Lambda_{\rm QCD}/m_c)^4$

TABLE I. Scaling of matrix elements relevant for ψ production in NRQCD$_b$ and NRQCD$_c$.

$$M_r^\psi = \langle O_8^\psi(^1S_0)\rangle + \frac{r}{m_c^2}\langle O_8^\psi(^3P_J)\rangle, \tag{4.2}$$

with $r \approx 3 - 3.5$, since the short-distance rates have similar size and shape. In the new power-counting, we can just drop the contribution from $\langle O_8^\psi(^3P_J)\rangle$, since it is down by $(\Lambda_{\rm QCD}/m_c)^2 \sim 1/10$ compared to $\langle O_8^\psi(^1S_0)\rangle$. It is the same order as $\langle O_8^\psi(^3S_1)\rangle$, but is not kinematically enhanced by fragmentation effects. The extraction from [27] would then give for the J/ψ and ψ' matrix elements

$$\begin{aligned}\langle O_8^{J/\psi}(^1S_0)\rangle : \langle O_8^{J/\psi}(^3S_1)\rangle &= (6.6\pm 0.7)\times 10^{-2} : (3.9\pm 0.7)\times 10^{-3} \approx 17:1\,,\\ \langle O_8^{\psi'}(^1S_0)\rangle : \langle O_8^{\psi'}(^3S_1)\rangle &= (7.8\pm 3.6)\times 10^{-3} : (3.7\pm 0.9)\times 10^{-3} \approx 2:1.\end{aligned} \tag{4.3}$$

Other extractions have various values of the hierarchy, ranging from $3:1$ to $20:1$ [28]. While the relation of the color-octet matrix elements in the J/ψ system is indeed in agreement with the NRQCD$_c$ power counting, the ψ' does not look to be hierarchical. However, it should be noted that the statistical errors in the ψ' extraction, quoted above, are quite large. Furthermore, there are also large uncertainties introduced in the parton distribution function. The above ratios used the CTEQ5L parton distribution functions. If we take the central values from [27] for the MRST98LO distribution functions, we find the ratio $3:1$. On the other hand, the J/ψ extraction is much less sensitive to the choice of distribution function. Given the statistical and theoretical errors, it clear that the ψ' ratio is not terribly illuminating.

Let us now consider the extraction of these color-octet matrix elements in the Υ sector [29], where according to NRQCD$_b$ power counting there is should be no hierarchy:

$$\begin{aligned}\langle O_8^{\Upsilon(3S)}(^1S_0)\rangle : \langle O_8^{\Upsilon(3S)}(^3S_1)\rangle &= (5.4\pm 4.3^{+3.1}_{-2.2})\times 10^{-2} : (3.6\pm 1.9^{+1.8}_{-1.3})\times 10^{-2} \approx 1:1, \quad (4.4)\\ \langle O_8^{\Upsilon(2S)}(^1S_0)\rangle : \langle O_8^{\Upsilon(2S)}(^3S_1)\rangle &= (-10.8\pm 9.7^{-3.4}_{+2.0})\times 10^{-2} : (16.4\pm 5.7^{+7.1}_{-5.1})\times 10^{-2} \approx 1:1,\\ \langle O_8^{\Upsilon(1S)}(^1S_0)\rangle : \langle O_8^{\Upsilon(1S)}(^3S_1)\rangle &= (13.6\pm 6.8^{+10.8}_{-7.5})\times 10^{-2} : (2.0\pm 4.1^{-0.6}_{+0.5})\times 10^{-2} \approx 6:1.\end{aligned}$$

For the $\Upsilon(3S)$ and $\Upsilon(2S)$ we observe that there is indeed no hierarchy, while for the $\Upsilon(1S)$ it appears like there may be a hierarchy [29]. However, it is not possible to draw any strong

	$\langle O_1^\chi(^3P_J)\rangle$	$\langle O_8^\chi(^3S_1)\rangle$	$\langle O_8^\chi(^1S_0)\rangle$	$\langle O_8^\chi(^3P_J)\rangle$	$\langle O_8^\chi(^1P_1)\rangle$
NRQCD$_b$	v^2	v^2	v^6	v^6	v^6
NRQCD$_c$	$(\Lambda_{\rm QCD}/m_c)^2$	$(\Lambda_{\rm QCD}/m_c)^2$	$(\Lambda_{\rm QCD}/m_c)^4$	$(\Lambda_{\rm QCD}/m_c)^6$	$(\Lambda_{\rm QCD}/m_c)^4$

TABLE II. Scaling of matrix elements relevant for χ production in NRQCD$_b$ and NRQCD$_c$.

conclusions from these data because the errors on the extractions are large. In fact the ratio for the $\Upsilon(1S)$ color-octet matrix elements is $1:1$ within the one sigma errors. Furthermore, these matrix elements are those extracted subtracting out the feed down from the higher states. Thus, the extraction of the $\Upsilon(1S)$ matrix elements actually have larger errors since the errors accumulate when we make the subtractions of the feed down components. Finally, we should add that, while phenomenologically it is perfectly reasonable to define the subtracted matrix elements, we believe that, since the matrix elements are inclusive, one should not subtract out the feed down from hadronic decays when checking the power counting. In principle this subtraction should not change things by orders of magnitude, but nonetheless can have a significant effect. Indeed, if one compares the ratios for inclusive matrix elements, which do not have the accumulated error, then the ratios come out to be $1:1$, even for the $\Upsilon(1S)$ [29].[8]

B. Fixed-target experiments

There are several phenomenological differences between NRQCD$_c$ and NRQCD$_b$ in fixed target experiments [31,32]. Here we will focus on ψ production and the predicted ratio of production cross sections $\sigma(\chi_1)/\sigma(\chi_2)$ in NRQCD$_c$.

At order α_s^2, ψs are produced via quark-antiquark fusion through the $\langle O_8^\psi(^3S_1)\rangle$ matrix element and through gluon fusion through the $\langle O_8^\psi(^1S_0)\rangle$ and $\langle O_8^\psi(^3P_0)\rangle$ matrix elements, in the linear combination M_7^ψ. At fixed-target energies, the contribution to ψ production from $\langle O_8^\psi(^3S_1)\rangle$ is numerically irrelevant because gluon fusion dominates. The difference between the NRQCD$_c$ prediction and the NRQCD$_b$ analysis done in [31] lies in the expected size of the matrix elements in M_7^ψ (called $\Delta_8(\psi)$ in [31]), since in NRQCD$_c$ the $^3P_0$ matrix element

[8]The authors of [30] have values of the extracted matrix elements that differ from [29]. They use PYTHIA to model initial state gluon radiation, and use data at small values of p_T in the extraction. Since we are worried about breakdown of factorization, we prefer to restrict our analysis to data points where factorization should hold.

is down by $(\Lambda_{\rm QCD}/m_c)^2$. However, since the $^3P_0$ matrix element is enhanced by a factor of 7, it is important to keep this formally subleading contribution. Furthermore, there is a very large scale and PDF dependence in these extractions, so it is not clear whether or not we can learn anything from comparisons with the Tevatron extractions.

The χ_1/χ_2 production ratio has the nice property that it is relatively insensitive to the charmed quark mass [31]. χ_1 production is suppressed as it can not be produced at leading order in the singlet channel. The formally leading order χ_1 channel is $^3S_1^{(8)}$ through quark-antiquark fusion. In both NRQCD$_b$ and NRQCD$_c$ this formally leading order contribution is actually smaller than the subleading contributions coming from other octet operators ($\langle O_8^{\chi_1}(^3P_0)\rangle$ and $\langle O_8^{\chi_1}(^3P_2)\rangle$ in NRQCD$_b$, and $\langle O_8^{\chi_1}(^1S_0)\rangle$ in NRQCD$_c$) due to the fact that these other channels are initiated by gluon-gluon fusion. The scalings for the χ matrix elements are shown in Table II. If we ignore the quark initiated process then due to simplicity of the $2 \to 1$ kinematics we may write the NRQCD$_b$ prediction as

$$\sigma_{\chi_1}/\sigma_{\chi_2} \simeq \frac{75}{32}\frac{\langle O_8^{\chi_1}(^1S_0)\rangle m_c^2 + 3\langle O_8^{\chi_1}(^3P_0)\rangle + 4O_8^{\chi_1}(^3P_2)\rangle}{\langle O_1^{\chi_2}(^3P_2)\rangle}\,, \tag{4.5}$$

where numerically small contributions have been dropped. This ratio is approximately 1/3 if we take take $v^2 \approx 0.3$. Also, the ratio is independent of the center of mass energy, which agrees with the data within errors [33].

In NRQCD$_c$ we have (again neglecting the numerically small quark-antiquark initiated processes)

$$\sigma_{\chi 1}/\sigma_{\chi_2} \simeq \frac{75}{32}\frac{\langle O_8^{\chi_1}(^1S_0)\rangle m_c^2}{\langle O_1^{\chi_2}(^3P_2)\rangle}\,, \tag{4.6}$$

where, once again, numerically small contributions have been dropped. If we take $\Lambda_{\rm QCD}/m_c \approx 1/3$ we get approximately the same result. This estimate is so crude that it is not clear whether any information can be gleaned from it. However, it does seem that in either description the data [34] is, on the average, larger than these naive predictions.[9] This could very well be due to large non-factorizable contributions, which we may expect to be enhanced in NRQCD$_c$ (see the conclusions).

V. POLARIZATION

J/ψ and ψ' are predicted to be transversely polarized at large p_T in NRQCD$_b$. At large transverse momentum, the dominant production mechanism is through fragmentation from a nearly on shell gluon to the octet $^3S_1$ state. The quark pair inherits the polarization of the fragmenting gluon, and is thus transversely polarized [7]. In NRQCD$_b$ the leading order transition to the final state goes via two $E1$, spin preserving, gluon emissions. Higher order perturbative fragmentation contributions [8], fusion diagrams [35,36], and feed-down for the

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[9]One robust prediction, however is that the ratio should be independent of s, which does seem to agree with the data.

J/ψ [27] dilute the polarization some, but the prediction still holds that as p_T increases so should the transverse polarization. Indeed, for the ψ', at large $p_T \gg m_c$, we expect nearly pure transverse polarization.

The polarization of J/ψ and ψ' at the Tevatron has recently been measured [2] with large error bars. The experimental results show no or a slight longitudinal polarization, as p_T increases. If, after the statistics improve, this trend continues, then it will be the smoking gun that leads us to conclude that NRQCD$_b$ is not the correct effective field theory for charmonia.

With NRQCD$_c$, the intermediate color-octet $^3S_1$ states hadronize through the emission of either two $E1$ or $M1$ dipole gluons, at the same order in $1/m_c$. Since the magnetic gluons do not preserve spin, the polarization of ψ produced through the $\langle O_8^\psi(^3S_1)\rangle$ can be greatly diluted. The net polarization will depend on the ratio of matrix elements

$$R_{M/E}= \frac{\int \prod_\ell d^4x_\ell \langle 0 \mid T(M_1(x_1)M_1(x_2)\psi^\dagger T^a \sigma_i \chi)\, a_H^\dagger\, a_H\, T(M_1(x_3)M_1(x_4)\chi^\dagger T^a\sigma_i\psi) \mid 0\rangle}{\int \prod_\ell d^4x_\ell \langle 0 \mid T(E_1(x_1)E_1(x_2)\psi^\dagger T^a \sigma_i \chi)\, a_H^\dagger\, a_H\, T(E_1(x_3)E_1(x_4)\chi^\dagger T^a\sigma_i\psi) \mid 0\rangle} \tag{5.1}$$

where

$$a_H^\dagger\, a_H = \sum_X \mid H + X\rangle\langle H + X \mid . \tag{5.2}$$

The leads to the polarization leveling off at large p_T at some value which is fixed by $R_{M/E}$. In Fig. 1, we show the prediction for J/ψ and ψ' polarization at the Tevatron. The data is from [2]. The three lines correspond to different values for $R_{M/E}$=(0 (dashed), 1 (dotted), ∞ (solid)). The dashed line is also the prediction for NRQCD$_b$. The residual transverse polarization for J/ψ at asymptotically large p_T is due to feed down from χ states. The non-perturbative corrections to our predictions are suppressed by $\Lambda^4_{\rm QCD}/m^4$.

VI. B DECAYS

Another useful observable for differentiating between NRQCD$_c$ and NRQCD$_b$ is charmonia production in B decays. Assuming perturbative factorization, the ψ production rate from semi-inclusive B decays may be written as

$$\Gamma(B \to H + X) = \sum_n C(b \to c\bar{c}[n] + x)\, \langle O_n^H\rangle. \tag{6.1}$$

This expression is valid up to power corrections of order $\Lambda_{\rm QCD}/m_{b,c}$, which parameterize the non-factorizable contributions. (To this accuracy it is justified to treat the B meson as a free b quark.) The short distance coefficients are determined by the $\Delta B = 1$ effective weak Hamiltonian

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} \left\{ V_{cb}^* V_{cq} \left[\frac{1}{3} C_{[1]}(\mu)\mathcal{O}_1(\mu) + C_{[8]}(\mu)\mathcal{O}_8(\mu)\right] - V_{tb}^* V_{tq} \sum_{i=3}^{6} C_i(\mu)\mathcal{O}_i(\mu) \right\} \tag{6.2}$$

containing the 'current-current' operators

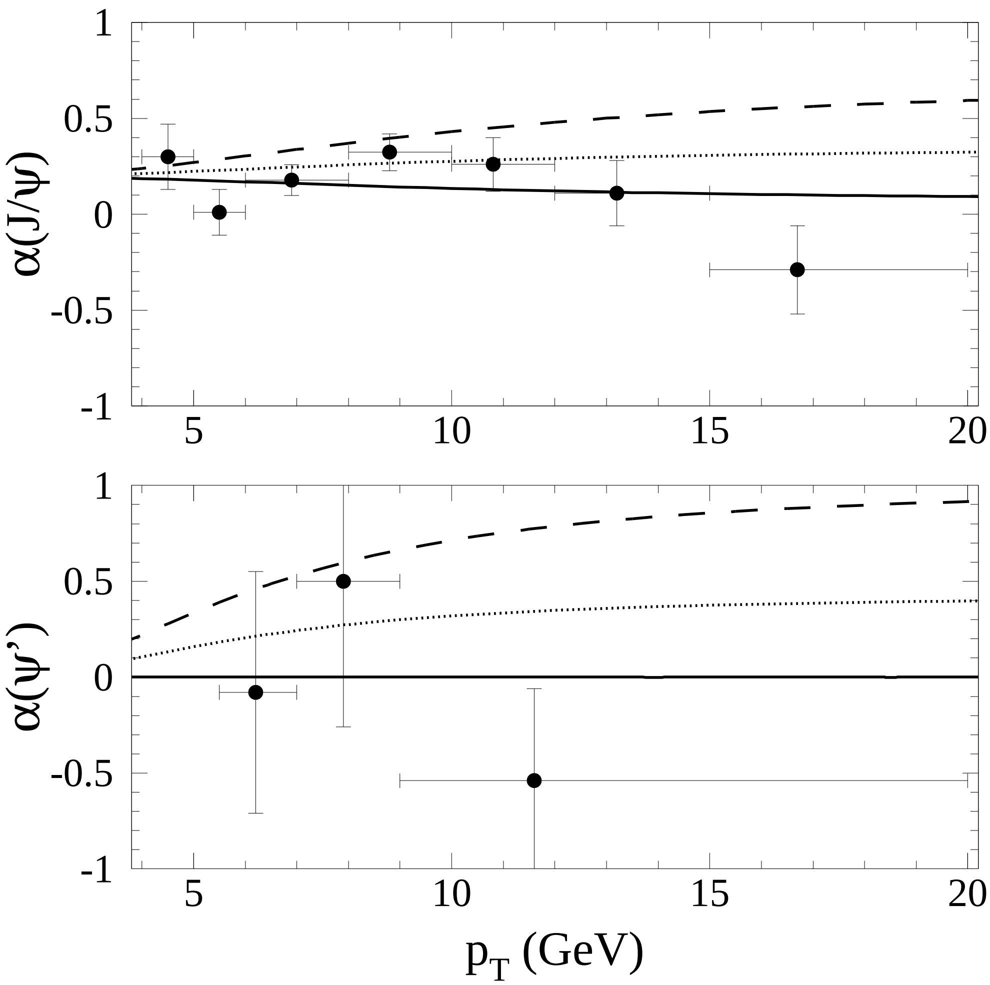

FIG. 1. Predicted polarization in NRQCD$_c$ for J/ψ and ψ' at the Tevatron as a function of p_T. The three lines correspond to $R_{M/E}$=(0 (dashed), 1 (dotted), ∞ (solid)). The dashed line is also the prediction for NRQCD$_b$.

$$\mathcal{O}_1 = [\bar{c}\gamma_\mu(1-\gamma_5)c]\,[\bar{b}\gamma^\mu(1-\gamma_5)q], \tag{6.3}$$

$$\mathcal{O}_8 = [\bar{c}\,T^A\gamma_\mu(1-\gamma_5)c]\,[\bar{b}\,T^A\gamma^\mu(1-\gamma_5)q], \tag{6.4}$$

and the QCD penguin operators $\mathcal{O}_{3-6}$ (precise definitions may be found in the review [37]). For the decays $B \to$ charmonium $+X$ it is convenient to choose a Fierz version of the current-current operators such that the $c\bar{c}$ pair at the weak decay vertex is either in a color-singlet or a color-octet state. The coefficient functions are related to the usual $C_\pm$ by

$$C_{[1]}(\mu) = 2C_+(\mu) - C_-(\mu), \tag{6.5}$$

$$C_{[8]}(\mu) = C_+(\mu) + C_-(\mu). \tag{6.6}$$

In NRQCD$_b$ a naive power counting leads to the conclusion that the leading order result is fixed by the $\langle O_1(^3S_1)\rangle$ operator as all octet operators are suppressed by v^4. However, as pointed out in Ref. [38], the fact that the Wilson coefficients evaluated at the low energy

scale are numerically hierarchical, $C_1^2/C_8^2 \approx 15$, actually leads to octet domination. In NRQCD$_b$ one would then get leading order contributions from all the octet matrix elements, $\langle O_8^{J/\psi}(^3S_1)\rangle$, $\langle O_8^{J/\psi}(^1S_0)\rangle$ and $\langle O_8^{J/\psi}(^3P_0)\rangle$, where the contribution from the other ^3P_J states have been written in terms of the $^3P_0$ contribution using spin symmetry.

In NRQCD$_c$, the leading order octet contribution comes solely from the $^1S_0$ operator which is suppressed by $\Lambda_{\rm QCD}^2/m_c^2$. Thus, we may get a direct extraction of this matrix element from the decay rate. However, at leading order in α_s the color-singlet contribution is highly scale dependent. This is due to large scale dependence in the value of $C_1(\mu)$. The authors of [31] found that the leading order singlet contribution varies by a factor of ten as μ is varied between 2.5 and 10 GeV. This scale dependence can be drastically reduced by working at next-to-leading order (NLO) and using a combined expansion in α_s and the ratio C_1/C_8 [39]. Using this expansion a NLO order calculation found that within NRQCD$_b$ power counting one could extract the linear combination [40]

$$M_{3.1}^{\psi}(^1S_0^{(8)}, {}^3P_J^{(8)}) = \begin{cases} 1.5\cdot 10^{-2}\,\mathrm{GeV}^3 & (J/\psi) \\ 0.6\cdot 10^{-2}\,\mathrm{GeV}^3 & (\psi'). \end{cases} \tag{6.7}$$

In NRQCD$_c$ the result is all spin singlet and we would thus conclude that $\langle O_8^{J/\psi}(^1S_0)\rangle = 1.5\cdot 10^{-2}$, which is quite a bit smaller than the Tevatron extraction .

We may also consider how the new power counting effects the prediction for the polarization of the ψ. In NRQCD$_b$ the prediction for the polarization, at leading order in α_s, was given in [41]. The angular distribution in the leptonic J/ψ decay may be written as

$$\frac{d\Gamma}{d\cos\theta}(\psi\to\mu^+\mu^-)(\theta) \propto 1+\alpha\,\cos^2\theta, \tag{6.8}$$

where the angle θ is defined in the J/ψ rest frame for which the z-axis is aligned with direction of motion of the J/ψ and

$$\alpha = \frac{\sigma(+)+\sigma(-)-2\sigma(0)}{\sigma(+)+\sigma(-)+2\sigma(0)}. \tag{6.9}$$

Within NRQCD$_b$ the authors found that α lies within the range $-0.4<\alpha<-0.1$ if the bottom quark mass lies between 4.4 and 5.0 GeV and the octet matrix elements are allowed to vary within their errors. This rather crude leading order prediction should be reasonable as long as the scale dependent singlet piece is not dominant, which it is not.

In NRQCD$_c$, given the color-octet $^1S_0$ dominance, we would expect a quenching of the polarization, since, as discussed in the case of hadro-production, the spin flipping hadronic transition involved in the matrix element obeys helicial democracy. Using the results of [41] we may write down the order α_s NRQCD$_c$ prediction for α

$$\alpha = \frac{-0.39\langle O_1^{J/\psi}(^3S_1)\rangle}{\langle O_1^{J/\psi}(^3S_1)\rangle + 61\langle O_8^{J/\psi}(^1S_0)\rangle}, \tag{6.10}$$

where we have kept the formally subleading $^1S_0$ contribution in the denominator because of its large coefficient. Since the singlet contribution to the polarization is now leading order, we need to be concerned about the scale dependence discussed above. Indeed, a

NLO calculation, in the modified double expansion scheme, is in order. If, as in the case of the polarized rate, the octet dominates, then we would expect only a slight longitudinal polarization. Note that the NRQCD$_c$ polarization prediction has the advantage that it only depends on one unknown matrix element, so once the NLO calculation has been done, the prediction will be comparatively robust.

VII. CONCLUSION

There are several relevant questions to ask. Is there any reason to believe that there is any effective theory to correctly describe the J/ψ? We believe that the spin symmetry predictions for the ratio of χ decays clearly answers this question in the affirmative. Assuming that such an effective theory exists, then is it NRQCD$_c$ or NRQCD$_b$? As we have shown the two theories do indeed make quite disparate predictions, which in principle should be easy to test. However, these tests can be clouded by the issues of factorization and the convergence of the perturbative expansion.

One would be justified to worry about the breakdown of factorization in hadro-production at small transverse momentum. Indeed, in NRQCD$_c$ where the time scale for quarkonia formation is assumed to be the same as the time scale for the hadronization of the remnants, it seems quite likely that there could be order one corrections to factorization.[10] Thus any support, or lack thereof, for the theory coming from these processes should be taken with a grain of salt. On the other hand, for large transverse momentum one would expect factorization to hold, with non-factorizable corrections suppressed by powers of m_c/p_T.

As far as the perturbative expansion is concerned, it seems that for most calculations the next-to-leading order results are indeed smaller than the leading order result [31,42,43]. However, the one NNLO calculation performed, in the leptonic decay width [44], is not well behaved at this order, which is worrisome. But, to truly test the convergence of the expansion we should take ratios of rates in order to eliminate the renormalon ambiguities [45]. When this is done, it could very well be that the perturbative expansion is well behaved. Until another rate is calculated at NNLO we will have to be comforted by the fact that such cancellations have been seen to occur explicitly in other heavy quark decays [46].

With that said, let us gather the evidence in support of NRQCD$_c$ as being the proper theory for the J/ψ. If one is willing to accept that the extraction of the octet matrix elements from CDF,[11] then the fact that the ratio is large for charmonia but seems to be small for bottomonia is rather compelling. If once the statistics in the bottomonia sector improve we find that there really is no hierarchy, then we believe that this would be strong evidence for our hypothesis. The fact that the J/ψ radiative decay spectrum is

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[10]This may be true as well in B decays. However, since most of the time the J/ψ will be going out back to back with the remnants, one might expect factorization to be more accurate.

[11]This extraction is not free of factorization issues since the fit of the matrix elements involves use of data at rather small values of transverse momentum. However, if a cut at $p_T = 5$ GeV is made, then the change in the fit is minimal.

monotonically decreasing while one sees a bump at larger energy in bottomonia also seems to lend credence to our hypothesis. However, the true litmus will come from the polarization measurements at large p_T. The predictions of nearly 100% polarization in NRQCD$_b$ is quite robust. Whereas, in NRQCD$_c$ the polarization is diluted from $M1$ transitions of the $O_8(^3S_1)$ operator. Unfortunately, the introduction of another unknown matrix element diminishes our predictive power. However, this is not to say that we can not rule out NRQCD$_c$. Indeed, NRQCD$_c$ also predicts a leveling off of the polarization with positive α. So a measurement of longitudinal (or, for J/ψ, zero) polarization would indeed negate our hypothesis.

We would like to close with a caveat. In particular, it should be pointed out that NRQCD$_c$ does not become exact in any limit. Typically, in an effective field theory, we expect that the ratio of sub-leading to leading contributions should vanish in a given limit of QCD. This gives us confidence that the theory MUST be correct in asymptotia. Whether or not the real world leads to a well behaved expansion though, is another question. For NRQCD$_c$ we might hope that as we take the limit $\Lambda_{\rm QCD}/m \to 0$, we necessarily get the correct answer. However, in this limit the soft modes become perturbative and the power counting changes. That is, in this limit the state becomes Coulombic and NRQCD$_b$ becomes the correct theory. It may well be the case that in some observables the NRQCD$_c$ expansion is well behaved and in others it is not. Given that the expansion parameter is around 1/3, it seems reasonable to be confident in those predictions for which the corrections are suppressed by at *least* $\Lambda^2_{\rm QCD}/m^2$ (modulo the convergence of the perturbative expansion), as are the predictions discussed in this paper.

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